Dependability Theory for Selection-Combined Channels with Rician Fading and Interference

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Abstract—The fifth generation (5G) of mobile networks aims to master the challenge of ultra-reliable low latency communications (URLLC). In this context, availability alone but combinations of availability and time-based multi-connectivity with selection-combined channels. Numerical downtimes (MDT), and availability are derived for single- and particular, analytic expressions for the mean uptime (MUT), mean interference are jointly considered as causes of failure. In particular, analytic expressions for the mean uptime (MUT), mean downtime (MDT), and availability are derived for single- and multi-connectivity with selection-combined channels. Numerical evaluations demonstrate the importance of analyzing not only availability alone but combinations of availability and time-based dependability statistics jointly.

Index Terms—5G, availability, dependability theory, URLLC, wireless networks

I. INTRODUCTION

The realization of wireless ultra-reliable low latency communications (URLLC) is a major challenge of the upcoming fifth generation (5G) of wireless communications systems and beyond. By ensuring latencies in the (sub-)millisecond range together with ultra-high reliability is expected to enable unprecedented use cases, e.g., wireless factory automation, self-driving cars, and real-time remote control, which pave the way towards the Tactile Internet [1], [2]. Within the context of 5G, the key performance indicator (KPI) reliability is considered as the percentage of successful transmissions, e.g., aiming for $1 - 10^{-5}$ up to $1 - 10^{-9}$ in URLLC [2], [3], [4]. However, by applying fundamental dependability theory, it is clear that this metric rather complies with the definition of availability [5]. In addition, it fails in reflecting any dependency on the time dimension. Although wireless communications systems suffer from signal fluctuations over time, which is called fading, there is a lack of analyzing time-based dependability attributes in the context of URLLC. The targeted optimum of zero mobility interruption is the only explicit time attribute in the current discussion, which of course cannot be guaranteed with 100% probability due to the random fading in wireless channels [6]. Only a few research activities have been performed aiming to leverage dependability theory to wireless channels with respect to time aspects: In [7], dependability theory concepts are applied and extended to wireless communications networks, which are modeled as repairable systems. Based on this work, a space-time approach was presented in [8]. Channel available and unavailable time intervals are modeled utilizing the channel occupancy status in [9], which is limited to cognitive radio networks. Since diversity is widely accepted to be key in order to compensate for fading, e.g., by sending messages redundantly over different wireless channels, we have recently determined closed-form expressions for mean up- and downtimes of a diversity system under Rayleigh fading [10]. In this paper, we extend our prior work by assuming the more general Rician fading, which allows to also cover channels including a dominant path, e.g., a line of sight (LoS). In addition, we jointly study two causes of failure, i.e., interference and small-scale fading. This work leverages fundamental dependability theory in order to propel the realization of wireless URLLC by refining the discussion on appropriate KPIs. We demonstrate the benefit of time-related KPIs, which remain almost unmentioned in wireless communications although they have been well accepted for many years in other areas such as electronic systems design. The main contributions of this paper are as follows:

- We apply concepts of dependability theory to the system design of wireless communications systems, i.e., how to
  - model a wireless communications system with Rician fading channels and interference as a system comprising repairable components, and
  - introduce redundancy by means of frequency diversity to improve dependability.

- Based on continuous-time Markov chains (CTMCs), we derive analytic expressions of the KPIs mean uptime (MUT), mean downtime (MDT) as well as availability for the considered scenario.

- We present the relations between the examined KPIs.

- We evaluate an exemplary scenario and numerically determine the fading margin, which minimizes the MDT.

II. SYSTEM MODEL

This paper is based on our prior work on applying dependability theory for wireless networks [10]. In this section,
we present the extended system model. The system can be described with existing terms from dependability theory by modelling channels to comprise repairable components. In dependability theory, it is often sufficient to distinguish between only two states: an operating state and a failed state. This applies to each component as well as to the system itself. The usual notion is that "up" is used for an component in an operational state whereas "down" refers to a failed state, i.e., in repair if repairable. Interpreting the wireless channel as a repairable item corresponds to the Gilbert-Elliot model [11]. The components can be affected by different causes of failure. In this paper, we address interference and small-scale fading as two major causes of failure for wireless communications.

A. Interference as a Cause of Failure

Co-channel interference is an important wireless access issue, e.g., for systems which operate in unlicensed frequency bands. We assume, that a user is not able to successfully transmit and receive messages via a channel, once an interferer begins to transmit over the same channel. Thus, we introduce the binary random variable interference state of channel $c_i$ according to

$$x_i^f(t) = \begin{cases} 0, & \text{if channel } c_i \text{ is interfered at } t, \text{ "down"} \\ 1, & \text{if channel } c_i \text{ is not interfered at } t, \text{ "up"} \end{cases}$$

with $i \in \mathbb{N}$. The interferer’s arrival rate on a channel $c_i$ is denoted by $\lambda_{i,1}$. The interferer’s service rate is described by $\mu_{i,1}$. From the user’s viewpoint the channel failure and repair rate with respect to interference correspond to $\lambda_{i,1}$ and $\mu_{i,1}$, respectively. We assume an equal probability for interferers to appear or leave any channel at all times, which leads to constant interference failure and repair rates, $\lambda_{i,1}(t) = \lambda_1$, $\mu_{i,1}(t) = \mu_1$. Every interference event is self-revealing, i.e., every state change is recognized immediately.

B. Rician Fading as a Cause of Failure

We concentrate on small-scale fading due to multi-path propagation as a further cause of failure for wireless communications systems. Presuming that each channel consists of many individual paths including a dominant component leads to a Rician fading channel [12]. A Rician faded signal can be successfully received if the instantaneous power $p_i(t)$ of a channel $c_i$ is above a certain threshold $p_{\text{min}}$, which may be determined by the receiver’s hardware sensitivity. We distinguish between two states by introducing the random variable fading state of channel $c_i$ according to

$$x_i^f(t) = \begin{cases} 0, & \text{if } p_i(t) < p_{\text{min}}, \text{ channel } c_i \text{ faded, "down"} \\ 1, & \text{if } p_i(t) \geq p_{\text{min}}, \text{ channel } c_i \text{ not faded, "up"} \end{cases}$$

assuming the same threshold $p_{\text{min}}$ for any channel. We assume that the random fading process is stationary. Thus, the transition rates between the two fading states, denoted as fading failure rate $\lambda_F$ and fading repair rate $\mu_F$, are constant and equal for any channel. These parameters can be specified by the reciprocals of the average (non-)fade duration of a Rician faded signal $\bar{\tau}^u$ and $\bar{\tau}^d$, respectively. The level crossing rate of a Rician fading signal yields

$$N_R = \frac{1}{\bar{\tau}^u + \bar{\tau}^d} = \sqrt{\frac{2\pi (K + 1)}{F}} f_D \exp \left(-K - \frac{1}{F}\right),$$

$$I_0 \left(2 \sqrt{\frac{K (K + 1)}{F}}\right),$$

where $F = \frac{p_{\text{avg}}}{p_{\text{min}}}$ represents the fading margin with the average receive power $p_{\text{avg}}$. $K$ is the Rician $K$-factor and $Q$ denotes the Marcum-Q function. The maximum Doppler frequency is characterized by $f_D = \frac{vf}{c}$, where $f$ is the carrier frequency of the signal and $c$ is the speed of light. The relative velocity between transmitter, receiver, and scatterers is denoted as $v$. By incorporating the known average fade duration $\bar{\tau}^d$ [12], we determine the fading failure and repair rate according to

$$\lambda_F = \frac{N_R}{\sqrt{2K} \sqrt{2(K + 1)}}$$

$$\mu_F = \frac{N_R}{1 - \sqrt{2K} \sqrt{2(K + 1)}}$$

Similar to the previous section, we assume that every fading is self-revealing, i.e., every state change is recognized immediately. In addition, compensation of path loss and shadowing by transmit power or automatic gain control is assumed. However, the proposed model can also be extended to wireless systems including path loss and shadowing, as well as other channel models.

C. Single Channel with Interference and Rician Fading

Subsequently, we jointly consider both causes of failure, interference and Rician fading. One channel $c_i$ is modelled to consist of two components: an interference stage $x_i^f$ and a fading stage $x_i^F$. The overall state of a single channel $c_i$ can be described by a binary function

$$x_i = \phi \left(x_i^f, x_i^F\right) = \begin{cases} 0, & \text{if channel } c_i \text{ is failed, "down"} \\ 1, & \text{if channel } c_i \text{ is operational, "up"} \end{cases}$$

We refer to $\phi \left(x_i^f, x_i^F\right)$ as the structure function of a single channel. If and only if messages can be successfully transmitted and received via a channel, the latter is assumed to be operational. Thus, it is obvious that the channel is up if and only if all components are up. This corresponds to a series structure with [13]

$$x_i = \phi \left(x_i^f, x_i^F\right) = x_i^f \cdot x_i^F.$$
a system with one channel has \( n = 4 \) states, which can be found in the first four columns of Table I. The state space is partitioned into the set of "up" states \( U_1 = \{3\} \) (highlighted green) and the set of "down" states \( D_1 = \{0, 1, 2\} \). The resulting CTMC is depicted in Fig. 1. The differential balance equations of this CTMC can be expressed in matrix terms as \( \dot{P}(t) = P(t) \cdot M_1 \) with the state probability row vector \( P(t) \), the state probability derivative vector \( \dot{P}(t) \), and the \( 4 \times 4 \) transition matrix

\[
M_1 = \begin{pmatrix}
-\lambda_1 - \lambda_F & \lambda_F & 0 & 0 \\
\mu_1 & -\mu_1 - \lambda_F & \lambda_F & 0 \\
\mu_F & 0 & -\mu_F - \lambda_1 & \lambda_1 \\
0 & \mu_F & \mu_1 & -\mu_F - \mu_1
\end{pmatrix}.
\] (8)

D. Two Channels with Interference and Rician Fading

Introducing redundant components is a standard approach to improve the dependability of a system. We focus on the case of two components, considering the system to be operational if and only if at least one of them is operational. This translates to wireless communications by assuming that the user’s communication is successful if at least one out of two wireless channels \( c_1, c_2 \) is operational. This diversity type is known as selection combining. The state of the whole system can be characterized by the binary structure function

\[
\theta(x_1, x_2) = \begin{cases} 
0, & \text{if the system is failed, "down"} \\
1, & \text{if the system is operational, "up"}
\end{cases}
\] (9)

with \( x_1, x_2 \) denoting the states of the two individual channels \( c_1, c_2 \), respectively. This translates to a parallel structure with [13]

\[
\theta(x_1, x_2) = 1 - (1 - x_1)(1 - x_2).
\] (10)

Inserting the structure function (7) of both channels \( c_1, c_2 \) with their interference and fading components \( x_1^1, x_1^2, x_2^1, x_2^2 \) yields

\[
\theta(\phi(x_1^1, x_1^2), \phi(x_2^1, x_2^2)) = 1 - (1 - x_1^1 x_1^2)(1 - x_2^1 x_2^2)
\] (11)

This can be illustrated by the reliability block diagram in Fig. 2 showing the logical connections of components necessary to fulfill the system function.

We extend the CTMC introduced in the previous section by adding the second channel. The finite system state \( j \) is defined as the decimal representation of \( (x_1^1, x_1^2, x_2^1, x_2^2) \) according to Table I. The system with two channels comprises \( n_2 = 16 \) states. The set of "up" states results in \( U_2 = \{3, 7, 11, 12, 13, 14, 15\} \) (highlighted green) and the set of "down" states is \( D_2 = \{0, 1, 2, 4, 5, 6, 8, 9, 10\} \). The obtained CTMC is shown in Fig. 3. For readability, the state transition variables are omitted. In analogy to Fig. 1, blue (red) transitions characterize interference (fading) state changes. Transitions to a higher (lower) system state \( j \) refer to a repair (failure). The probability that more than one channel enters or leaves the failed state at exactly the same time is negligible.

III. DEPENDABILITY METRICS

 Dependability theory is a powerful framework, involving the main attributes availability, reliability, maintainability, safety, integrity, and security. In this section, we introduce basic dependability quantities and apply them to the considered wireless communications system, which is modeled as a repairable system corresponding to Section II.
A. Availability

According to [14], "an item is available, if it is in a state to perform a required function at a given instant of time or at any instant of time within a given time interval, assuming that the external resources, if required, are provided." An item refers to a component as well as to the whole system. The dependability metric availability is denoted by \( A \) characterizing the long-term probability of a component to be operational [13]. Thus, we derive the following availability quantities for the introduced components modelling the wireless channel:

\[
A_i^I = A^I_i = \Pr \{ x_i^I \}, \quad (12)
\]
\[
A_i^F = A^F_i = \Pr \{ x_i^F \}, \quad (13)
\]
\[
A_1 = \Pr \{ x_1 \} = \Pr \{ x_1^I \wedge x_1^F \} = A^I_1 A^F, \quad (14)
\]
\[
A_2 = \Pr \{ x_1 \wedge x_2 \} = 1 - \Pr \{ \bar{x}_1 \wedge \bar{x}_2 \} = 1 - (1 - A_1)^2 = 1 - (1 - A^I_1 A^F)^2 \quad (15)
\]

with \( i \in \{1, 2\} \), assuming the same statistics for both channels. \( A_1 \) denotes the availability of one channel, whereby \( A_2 \) characterizes the availability of the whole system comprising two parallel channels. Since availability corresponds to the mean proportion of time a component (or system) is operational, we derive the availability of the fading component according to

\[
A^F = \frac{x^u}{x^u + x^d} = \frac{\mu_F}{\mu_F + \lambda_F} = Q \left( \sqrt{2K}, \sqrt{\frac{2(K+1)}{F}} \right), \quad (16)
\]

which confirms the known expression for Rician fading [12]. Please note that, for the considered wireless communications system scenario, this fading availability purely depends on the fading margin \( F \) and the Rician \( K \)-factor. Hence, this metric does not reflect the influence of mobility aspects or the carrier frequency on the communication performance. Similarly, the availability of the component modelling interference yields

\[
A^I = \frac{\mu_1}{\mu_1 + \lambda_1} = \frac{1}{1 + \frac{\lambda_1}{\mu_1}}. \quad (17)
\]

Obviously, all interference failure and repair rates with the same ratio lead to the same interference availability value \( A^I \).

We can analytically determine the availability for one and two channels, \( A_1, A_2 \), by utilizing the derived CTMCs, too. The considered wireless communications system is available if it is in one of the system up states aggregated in \( U \) and \( \bar{U} \), respectively. The availability can be determined as

\[
A_k = \sum_{j \in U_k} P_{k,j}. \quad (18)
\]

The steady-state probabilities \( P_k = [P_{k,0}, P_{k,1}, \ldots, P_{k,n_k-1}] \) satisfy the matrix equation \( P_k \cdot M_k = [0, 0, \ldots, 0] \) with \( k \in \{1, 2\} \). The resulting availability expressions confirm equations (14) and (15). The complement of the steady-state channel availability \( A_{k,n} \) characterizes the outage probability given by

\[
A^{out,k} = 1 - A_k = \left( 1 - A^I_1 A^F \right)^k. \quad (19)
\]

It corresponds to the packet loss rate (PLR), a quantity often used to specify dependability requirements in communications systems, because it can be interpreted as the long-term probability that the communications system is not operational.

B. Mean Downtime

The mean downtime (MDT) identifies the mean duration of a system failure, defined as the mean time from when the system enters a down state until it is repaired and transitions back to an up state [13]. This is an essential metric to draw conclusions about a system’s capability to self-repair/recover after a failure. Based on the frequency of system failures

\[
\omega_k = \sum_{i \in U_k} \sum_{\ell \in \bar{U}_k} P_{k,i} \cdot a_{k,i,\ell} \quad (20)
\]

the MDT\(_k\) for a user with \( k \in \{1, 2\} \) channels can be calculated as

\[
\text{MDT}_k = \frac{1 - A_k}{\omega_k}, \quad (21)
\]

where \( a_{k,i,\ell} \) denotes the transition rate from state \( i \) to \( \ell \) in a system with \( k \) parallel channels [10]. Applied to the introduced scenario, the mean downtime for a user with \( k \in \{1, 2\} \) channel(s) results in

\[
\text{MDT}_k = \frac{1 - A^I_1 A^F}{k A^I_1 A^F (\lambda_1 + \lambda_F)}. \quad (22)
\]

The MDT linearly scales with the number \( k \) of parallel channels because this metric solely considers the system’s downtime, i.e., all channels are failed and any single channel repair leads to a system repair.

C. Mean Uptime

The mean uptime (MUT) characterizes the mean system operational time until a failure occurs. Regarding the considered scenario, it is defined as the mean time from a transition to an up state until the first transition back to a down state. Using the relation [10]

\[
\text{MUT}_k = \frac{A_k}{\omega_k}, \quad (23)
\]

we obtain

\[
\text{MUT}_1 = \frac{1}{\lambda_1 + \lambda_F}, \quad (24)
\]
\[
\text{MUT}_2 = \frac{2 - A^I_1 A^F}{2(1 - A^I_1 A^F)(\lambda_1 + \lambda_F)} \quad (25)
\]

as the mean uptime for a user with \( k \in \{1, 2\} \) channel(s), respectively. In contrast to the metric availability \( A_k \), the user’s MDT\(_k\) and MUT\(_k\) depend on the fading margin \( F \), Rician \( K \)-factor, and maximum Doppler shift \( f_{\Delta} \), included in \( \lambda_F \), as well as the interference failure rate \( \lambda_1 \) and the interference availability \( A^I \). Hence, we propose to utilize these KPIs for the research on wireless communications systems, because these metrics allow to evaluate the dependability of a wireless communications system from the user’s viewpoint taking into account all actual rates \( \lambda_1, \lambda_F, \mu_1, \) and \( \mu_F \). It is obvious...
that system dependability is based upon the quantities MUT\(_k\), MDT\(_k\), besides availability \(A_k\), which can be linked by the fundamental relation

\[ A_k = \frac{\text{MUT}_k}{\text{MUT}_k + \text{MDT}_k}. \]

Please note that the special case of Rayleigh fading (\(K = 0\)) without interference (\(\lambda_1 = 0\)) leads to the dependability metric expressions determined in our prior work [10].

**IV. Evaluation Scenario and Results**

In this section, the considered dependability metrics are evaluated for the exemplary scenario comprising medium velocity \(v = 10 \text{ m/s}^{-1}\) and the carrier frequency \(f = 2 \text{ GHz}\), equivalent to \(f_D = 66.6 \text{ Hz}\). We compare the performance for different Rician \(K\)-factors and interference outage probabilities \(P_{\text{out}}^{1} = 1 - A^1\) with the exemplary interference repair rate \(\mu_1 = 0.1 \text{ Hz}\), if not stated otherwise.

Fig. 4 depicts the outage probability \(P_{\text{out},k}\) (or system unavailability) for \(k \in \{1, 2\}\) channel(s). Adding a parallel channel leads to exponentially improved system availability, \(P_{\text{out},2} = P_{\text{out},1}^{2}\), which follows from equations (14) and (15). Thus, both cases are presented in one plot, utilizing two axes. The outage probabilities decrease with higher fading margins \(F\). The overall availability within the series structure of a single channel is limited by the component (interference or fading) which exhibits the higher outage probability. This is clearly visible because \(P_{\text{out},1}\) approaches \(P_{\text{out}}^{1}\) for increasing fading margin \(F\). Higher Rician \(K\)-factors, which indicate more dominant LoS components in the signal, can reduce the system outage probability by several orders of magnitude. The gain increases for larger fading margins \(F\).

Equivalently, this translates to a gain in terms of fading margin for a given outage probability: Comparing \(K = 7 \text{ dB}\) and \(K = 14 \text{ dB}\) corresponds to a fading margin gain of \(12 \text{ dB}\) for \(P_{\text{out},2} = 10^{-6}\), which is considered sufficient for many URLLC applications [2]. However, it is obvious that this KPI fails to reflect the performance with respect to time-related aspects, e.g., time-varying channels or the duration of an interference state in a wireless system: For instance, an availability value of 99% may refer to frequent failures (e.g., on average every 99 ms) and short downtimes (1 ms) as well as long downtimes (e.g., 10 min) and relatively rare failures (one failure every 16.5 h). The consequences for a real system are of course fundamentally different. Thus, it is beneficial to define requirements taking into account the duration of uptime or downtime.

Fig. 5 and Fig. 6 illustrate the MUT\(_k\) for \(k = 1, 2\) channel(s), respectively. The MUT increases for larger fading margin \(F\), Rician \(K\)-factor, number \(k\) of parallel channels, and lower interference outage probability \(P_{\text{out}}^{1}\). The highest gain is visible for large fading margins \(F\): For instance, comparing \(k = 1\) Rayleigh faded (\(K = 0\)) channel with \(k = 2\) parallel channels exhibiting a clear LoS (Rician fading with \(K = 14 \text{ dB}\)) leads to an improvement from MUT\(_1 = 100 \text{ ms}\) to MUT\(_2 > 15 \text{ years}\) at \(F = 20 \text{ dB}\) and \(P_{\text{out}}^{1} = 10^{-4}\).
V. Conclusion

In this paper, we applied fundamental dependability theory to a wireless communications scenario in order to demonstrate that the consideration of time-based KPIs is of great benefit for designing wireless communications systems, especially regarding URLLC. Based on CTMCs, we obtained analytic expressions of the availability and MUT, MDT, focusing on selection-combined Rician fading channels with interference. Supported by numerical examples, we discussed the trade-off between fading margin, Rician $K$-factor, and interference statistics for single- and dual-connectivity. These investigations are basic steps to refine the discussion on URLLC, which will help mastering key challenges of future wireless communications systems.

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